Name: KEY **Homework 4, GEL 161**

Ch. 7 [35 pts]

1. The cross-over distance is the distance at which the travel time for a direct and refracted wave have the same arrival time at a single receiver. Derive an equation for the cross-over distance, x_{crs}, from the travel time equations for a direct and refracted wave. [5 pts]
   The cross-over distance is defined as the source-receiver distance at which the direct and refracted rays arrive simultaneously. Therefore their travel times are equal:
   $t_{dir} = t_{refr}$
   $\frac{x_{crs}}{v_1} = \frac{x_{crs}}{v_2} + \frac{2z\cos(\theta_c)}{v_1}$ Solve for x_{crs}
   $x_{crs} = 2z\cos(\theta_c)(\frac{v_2}{v_2 - v_1})$

2. The first arrival of a zero-offset reflection event at 1.000 s is recorded at a receiver 250 m away at 1.008 s. What is the layer velocity? [5 pts]
   Given:
   $t_0 = 1.000$ s, $t_x = 1.008$ s, and $x = 250$ m
   Use normal move-out equation:
   $\Delta T = t_x - t_0 = \frac{x^2}{2V^2 t_0}$
   Solve for V:
   $V = \sqrt{\frac{x^2}{2\Delta T t_0}}$
   $V = \sqrt{\frac{250^2}{2\times 0.008\times 1.000}}$
   $V = 1976$ m/s

3. The arrival times for reflections from three interfaces are recorded at two stations. Station 1 is located at the same location as the source ($x = 0$) and station 2 is located at a distance of $x = 1040$ m. The recorded arrival times at each station are given in the table below (in seconds) [25 pts]

Station 1 ($x = 0$)	Station 2 ($x = 1040$ m)
0.2069	0.4136
0.5069	0.5831
0.6607	0.7090

   (a) Draw a schematic illustrating the source, receiver, three interfaces and the ray paths for all the arrivals at station 1 and station 2.
   See figure 7.6.

   (b) Determine the interval velocity and layer thickness for all three layers corresponding to the arrival times from the three interfaces.
   Step 1: Calculate normal move-out times for each reflection, $\Delta T_i = t_x - t_o$:
   $\Delta T_1 = 0.2067$
   $\Delta T_2 = 0.0762$
   $\Delta T_3 = 0.0483$

Step 2: Calculate the interval velocity for layer 1:
$v_1 = \sqrt{x^2/(t_1^2 + t_{o,1}^2)}$, $v_1 = 2904$ m/s

Step 3: Calculate rms-velocity for layers 2 and 3, $V_{rms,n} = x/\sqrt{2 * t_{o,n} * \Delta T_n}$:
$V_{rms,2} = x/\sqrt{2 * t_{o,2} * \Delta T_2}$, $V_{rms,2} = 3741.8$ m/s
$V_{rms,3} = x/\sqrt{2 * t_{o,3} * \Delta T_3}$, $V_{rms,3} = 4116.6$ m/s

Step 4: Uses Dix's Formula to calculate layer 2 and 3 interval velocity,
$v_n = \sqrt{\frac{V_{rms,n}^2 t_n - V_{rms,n-1}^2 t_{n-1}}{t_n - t_{n-1}}}$:

Layer 2: $v_2 = \sqrt{\frac{V_{rms,2}^2 t_2 - v_1^2 t_1}{t_2 - t_1}}$, $v_2 = 4160$ m/s

Layer 3: $v_3 = \sqrt{\frac{V_{rms,3}^2 t_3 - V_{rms,2}^2 t_2}{t_3 - t_3}}$, $v_3 = 5531$ m/s

Step 5: Calculate the layer thicknesses, using $z_n = v_n * t_n/2$, where t_n is the travel-time in a particular layer:

Layer 1: $z_1 = v_1 * t_{o,1}/2$
$z_1 = 300$ m
Layer 2: $z_2 = v_2 * (t_{o,2} - t_{o,1})/2$
$z_2 = 624$ m
Layer 3: $z_3 = v_3 * (t_{o,3} - t_{o,2})/2$
$z_3 = 425$ m

(c) Compare your results to the actual structure, which is $v_1 = 2900$ m/s, $v_2 = 4000$ m/s and $v_3 = 5200$ m/s and $z_1 = 300$ m, $z_2 = 600$ m, and $z_3 = 400$ m. Are your estimates of the interval velocities and layer thicknesses good estimates (use percentage error to quantify)?Are all the values equally good or bad estimates? Explain your answer.
Layer 1 has a small error, because you can use the exact solution for v_1. Dix's formula is used to calculate the layer 2 and 3 velocities, which leads to fairly small errors.

v_1 percentage error: 0.14% z_1 percentage error: 0.13%
v_2 percentage error: 4.0%, z_2 percentage error: 4.0%
v_3 percentage error: 6.25%, z_3 percentage error: 6.25%

The layer thickness should be estimated by using v_1, v_2, and v_3, but note that for the deeper layers, the appropriate time to use is the time spent in that layer. Alternatively, for the deeper layers one could use $V_{rms,n}$ and then subtract out the thickness of the shallower layers. Both approaches give similar error, the difference is that using the interval velocities leads to an over-estimation of layer thickness, while using the RMS-velocity tends to under-estimate the layer thickness, with slightly smaller errors.